1 Bridging Unit: Algebra 1

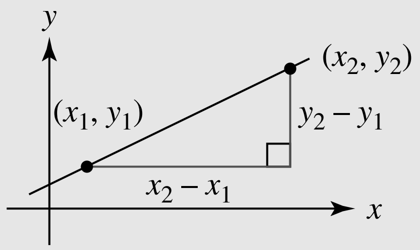
Topic F: Line graphs

Bridging to Ch1.5

This topic recaps how you can calculate key properties of straight line graphs when given two points on the line, in particular: the gradient, the length of a line segment, the midpoint of a line segment, the equation of the perpendicular bisector of a line segment, and the equation of the line. The gradient of a line is a measure of how steep it is.

Key point

The gradient, m, of a line between two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$



Example 1

Calculate the gradient of the line through the points $A(1, -6)$ and $B(-5, 2)$

$$m = \frac{2-(-6)}{(-5)-1}$$

Use $m = \frac{y_2 - y_1}{x_2 - x_1}$ with $x_1 = 1$, $x_2 = -5$ and $y_1 = -6$, $y_2 = 2$

$$= \frac{8}{-6}$$

$$= -\frac{4}{3}$$

The line has a negative gradient so slopes down from left to right.

Try It 1

Find the gradient of the line through each pair of points.

a (1, 7) and (4, 8) **b** (8, –2) and (4, 6) **c** (–8, 7) and (–4, –7)

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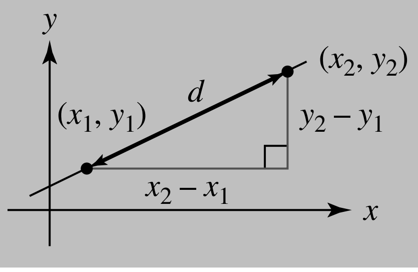
You also can find the length of a line segment between two points using Pythagoras' theorem.

Key point

The length of the line segment, d, between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Example 2

Calculate the exact distance between the point (5, 1) and (6, –4)

$d = \sqrt{(6-5)^2 + (-4-1)^2}$

$= \sqrt{1^2 + (-5)^2}$

$= \sqrt{26}$

Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ with $x_1 = 5$, $x_2 = 6$ and $y_1 = 1$, $y_2 = -4$

Leave answer as a surd since this is **exact**.

Try It 2

Calculate the exact distance between each pair of points.

a (5, 2) and (7, 4) **b** (6,–4) and (–3, –1) **c** $(\sqrt{2}, 4)$ and $(4\sqrt{2}, -5)$

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The midpoint of a line segment is half-way between the points at either end.

Key point

The midpoint of the line segment from (x_1, y_1) to (x_2, y_2) is $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

Example 3

The points *A* and *B* have coordinates (–4, –9) and (6, –2) respectively. Find the midpoint of *AB*

$$\text{Midpoint} = \left(\frac{(-4)+6}{2}, \frac{(-9)+(-2)}{2}\right)$$
$$= \left(\frac{2}{2}, \frac{-11}{2}\right)$$
$$= (1, -5.5)$$

Use $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$ with $x_1 = -4$, $x_2 = 6$ and $y_1 = -9$, $y_2 = -2$

Try It 3

Calculate the midpoint of the line segment between each pair of points.

a (1, 9) and (2, 5) **b** (–2, 3) and (–5, –7) **c** (6.4, –9.3) and (–2.6, –3.7)

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You need to be able to work with equations of straight lines.

Key point

The equation of a straight line is $y = mx + c$ where m is the gradient and c is the y-intercept.

Example 4

Work out the gradient and the y-intercept of each of these lines.

a $y = \frac{1}{2}x + 4$ **b** $y + x = 5$ **c** $-2x + 3y + 7 = 0$

a Gradient $= \frac{1}{2}$ and y-intercept $= 4$

Since $y = mx + c$ where m is the gradient and c is the y-intercept.

b $y = 5 - x$

Rearrange to make y the subject.

So gradient $= -1$ and y-intercept $= 5$

c $3y = -7 + 2x$

Rearrange to make y the subject.

$y = -\frac{7}{3} + \frac{2}{3}x$

So gradient $= \frac{2}{3}$ and y-intercept $= -\frac{7}{3}$

Try It 4

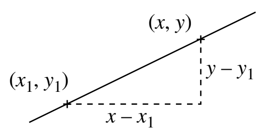
Work out the gradient and the y-intercept of each line.

a $y = 8 - 2x$ **b** $2y + x = 3$ **c** $6x - 9y - 4 = 0$

You can write the gradient of a line in terms of a known point on the line (x_1, y_1), the general point (x, y), and the gradient, m.

$m = \frac{y - y_1}{x - x_1}$ or alternatively $y - y_1 = m(x - x_1)$

Gradient $= m = \frac{y - y_1}{x - x_1}$



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Key point

The equation of the line with gradient m through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

If you have the coordinates of two points on a line then you can find the equation of the line. First use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line then substitute into $y - y_1 = m(x - x_1)$. Sometimes you will then need to rearrange the equation into a specific form.

Example 5

Find the equation of the line through the points (3, 7) and (4, –2) in the form $y = mx + c$

$m = \frac{(-2)-7}{4-3}$

$= -9$

So the equation is $y - 7 = -9(x - 3)$

$y - 7 = -9x + 27$

$y = -9x + 34$

First use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient.

Use $y - y_1 = m(x - x_1)$ with $(x_1, y_1) = (3, 7)$, or you could use the point (4, –2) instead.

Expand the brackets and rearrange to the correct form.

Try It 5

Find the equation of the line through each pair of points.

a (3, 7) and (2, 9) **b** (5, –1) and (7, 5) **c** (–3, –4) and (7, 2)

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Lines with the same gradient are **parallel**. For example, $y = 5x + 2$ is parallel to $y = 5x - 7$, because the gradients are the same.

Example 6

The line l_1 has equation $2x + 6y = 5$. The line l_2 is parallel to l_1 and passes through the point (1, –5). Find the equation of l_2 in the form $ax + by + c = 0$ where a, b and c are integers.

$l_1: 2x + 6y = 5 \Rightarrow 6y = 5 - 2x$

$\Rightarrow y = \frac{5}{6} - \frac{2}{6}x$ — Rearrange to make y the subject so you can see what the gradient is.

The gradient of l_1 is $-\frac{2}{6}$ which simplifies to $-\frac{1}{3}$

Therefore the gradient of l_2 is $-\frac{1}{3}$ — Since l_1 and l_2 are parallel.

So the equation of l_2 is $y - (-5) = -\frac{1}{3}(x - 1)$ — Use $y - y_1 = m(x - x_1)$ to write the equation of l_2

$\Rightarrow y + 5 = -\frac{1}{3}(x - 1)$

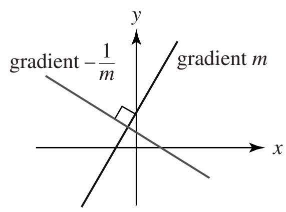
$\Rightarrow -3y - 15 = x - 1$ — Multiply both sides by –3 so that all coefficients are integers.

$\Rightarrow x + 3y + 14 = 0$ — Rearrange to the correct form.

Try It 6

The line l_1 has equation $3x - 2y = 8$. A second line, l_2 is parallel to l_1 and passes through the point (3, –2). Find the equation of l_2 in the form $ax + by + c = 0$ where a, b and c are integers.

Lines that meet at a right angle are **perpendicular**. The gradients of two perpendicular lines multiply to give –1. For example, a line with gradient 5 is perpendicular to a line with gradient $-\frac{1}{5}$ since $5\times\left(-\frac{1}{5}\right)=-1$



Key point

If the gradient of a line is m then the gradient of a perpendicular line is $-\frac{1}{m}$ since $m\times\left(-\frac{1}{m}\right)=-1$

Example 7

Decide whether or not each line is parallel or perpendicular to the line $y=4x-1$

a $2x+8y=5$ **b** $20x+5y=2$ **c** $16x-4y=5$

First note that the gradient of $y=4x-1$ is 4

a $2x+8y=5 \Rightarrow 8y=5-2x$ — Rearrange to make y the subject.

$\Rightarrow y=\frac{5}{8}-\frac{1}{4}x$ — The gradient is $-\frac{1}{4}$

$4\times\left(-\frac{1}{4}\right)=-1$ so this line is perpendicular to $y=4x-1$ — Since the product of the gradients is –1

b $20x+5y=2 \Rightarrow 5y=2-20x$

$\Rightarrow y=\frac{2}{5}-4x$ — Rearrange to make y the subject.

The gradient is –4 so this line is neither parallel nor perpendicular to $y=4x-1$

c $16x-4y=5 \Rightarrow 4y=16x-5$

$\Rightarrow y=4x-\frac{5}{4}$

The gradient is 4 so this line is parallel to $y=4x-1$

Try It 7

Decide whether or not each line is parallel or perpendicular to the line $y=4-3x$

a $3x+6y=2$ **b** $5x-15y=7$ **c** $18x+6y+5=0$

Example 8

The line l_1 has equation $7x+4y=8$ The line l_2 is perpendicular to l_1 and passes through the point (7, 3). Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

$l_1: 7x+4y=8 \Rightarrow 4y=-7x+8$

$\Rightarrow y=-\frac{7}{4}x+2$ — Rearrange to make y the subject so you can see what the gradient is.

So the gradient of l_1 is $-\frac{7}{4}$ and the gradient of l_2 is $\frac{4}{7}$ — Since $\left(-\frac{7}{4}\right)\times\frac{4}{7}=-1$

So the equation of l_2 is $y-3=\frac{4}{7}(x-7)$ — Use $y-y_1=m(x-x_1)$ to write the equation of l_2

$\Rightarrow 7y-21=4(x-7)$ — Multiply both sides by 7 so that all coefficients are integers.

$\Rightarrow 7y-21=4x-28$

$\Rightarrow 4x-7y-7=0$ — Rearrange to the correct form.

Try It 8

The line l_1 has equation $4x+6y=3$. A second line, l_2 is perpendicular to l_1 and passes through the point (–1, 5). Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

The **perpendicular bisector** of a line segment passes through its midpoint at a right angle.

Example 9

Find the equation of the perpendicular bisector of the line segment joining (3, –4) and (9, –6)

Midpoint is $\left(\frac{3+9}{2}, \frac{-4+(-6)}{2}\right) = (6, -5)$

Use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Gradient of line segment is $\frac{-6-(-4)}{9-3} = -\frac{2}{6} = -\frac{1}{3}$

Use $m = \frac{y_2-y_1}{x_2-x_1}$

So the perpendicular bisector has gradient $m = 3$

Since they are perpendicular and $3 \times \left(-\frac{1}{3}\right) = -1$

The equation of the perpendicular bisector is $y-(-5) = 3(x-6)$

Use $y - y_1 = m(x - x_1)$

or $y = 3x - 23$

Try It 9

Find the equation of the perpendicular bisector of the line segment joining (2, –3) and (–12, 5)

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Bridging Exercise Topic F

Bridging to Ch1.5

1 Find the gradient of the line through each pair of points.

a $(3, 7)$ and $(2, 8)$

b $(5, 2)$ and $(-4, -6)$

c $(1.3, 4.7)$ and $(2.6, -3.1)$

d $\left(\frac{1}{2}, \frac{1}{3}\right)$ and $\left(\frac{3}{4}, \frac{2}{3}\right)$

e $(\sqrt{3}, 2)$ and $(2\sqrt{3}, 5)$

f $(3a, a)$ and $(a, 5a)$

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2 Calculate the exact distance between each pair of points.

a (8, 4) and (1, 3)

b (– 3, 9) and (12, –7)

c (5.9 ,6.2) and (–8.1, 3.8)

d $\left(\frac{1}{5},-\frac{1}{5}\right)$ and $\left(\frac{3}{5},-\frac{4}{5}\right)$

e $(5, -3\sqrt{2})$ and $(2, \sqrt{2})$

f $(k, -3k)$ and $(2k, -6k)$

3 Find the coordinates of the midpoint of each pair of points.

a (3, 9) and (1, 7)

b (2, –4) and (–3, –9)

c (2.1, 3.5) and (6.3, –3.7)

d $\left(\frac{2}{3}, -\frac{1}{2}\right)$ and $\left(-\frac{5}{3}, -\frac{3}{2}\right)$

e $(6\sqrt{5}, 2\sqrt{5})$ and $(-\sqrt{5}, \sqrt{5})$

f $(m, 2n)$ and $(3m, -2n)$

4 Work out the gradient and the y-intercept of these lines.

a $y = 7x - 4$

b $y + 2x = 3$

c $x - y = 4$

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d $3x+2y=7$

e $5x-2y=9$

f $5y-3x=0$

g $x+6y+3=0$

h $3(y-2)=4(x-1)$

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5 Find the equation of the line through each pair of points.

a (2, 5) and (0, 6)

b (1, –3) and (2, –5)

c (4, 4) and (7, –7)

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d (8, −2) and (4, −3)

e (−3, −7) and (5, 9)

f $(\sqrt{2}, -\sqrt{2})$ and $\left(3\sqrt{2}, 4\sqrt{2}\right)$

6 Which of these lines is either parallel or perpendicular to the line with equation $y = 6x + 5$?

a $2x + 12y + 3 = 0$

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b $18x + 3y = 2$

c $3x - \frac{1}{2}y + 5 = 0$

7 Which of these lines is either parallel or perpendicular to the line with equation $y = \frac{2}{3}x - 4$?

a $24x + 16y + 3 = 0$

b $6x + 9y + 2 = 0$

c $2x - 3y = 7$

8 Which of these lines is either parallel or perpendicular to the line with equation $6x+12y=1$?

a $2y=5-x$

b $9x=18y+4$

c $10x-5y+3=0$

In questions **9–13**, give your answers in the form $ax+by+c=0$ where a, b and c are integers.

9 The line l_1 has equation $y=5x+1$

a Find the equation of the line l_2 which is parallel to l_1 and passes through (3, –3)

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b Find the equation of the line l_2 which is perpendicular to l_1 and passes through (–4, 1)

10 The line l_1 has equation $y = 3 + \frac{1}{2}x$

a Find the equation of the line l_2 which is parallel to l_1 and passes through (–1, 5)

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through (6, 2)

11 The line l_1 has equation $3x + y = 9$

a Find the equation of the line l_2 which is parallel to l_1 and passes through (8, –2)

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b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $(-1, -1)$

12 The line l_1 has equation $6x+5y+2=0$

a Find the equation of the line l_2 which is parallel to l_1 and passes through $(4, 0)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $(12, 3)$

13 The line l_1 has equation $6x-2y=1$

a Find the equation of the line l_2 which is parallel to l_1 and passes through $\left(\frac{1}{2}, 1\right)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $\left(-1, -\frac{1}{2}\right)$

14 Find the equation of the perpendicular bisector of the line segment joining each pair of points.

a (5, –7) and (–3, 5)

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b (–5, –9) and (5, 5)

c (–6, 2) and (4, 12)

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d (2, –7) and (–1, 2)

e (–13, –5) and (15, –12)

15 Find the point of intersection between these pairs of lines.

a $y = 5x - 4$ and $y = 3 - 2x$

b $y=8x$ and $y=3x-10$

c $y=7x-5$ and $y=-\frac{1}{2}x+5$

d $y = \frac{1}{4}x + 7$ and $y = 5x - \frac{5}{2}$

16 Find the point of intersection between these pairs of lines.

a $2x + 3y = 1$ and $3x - y = 7$

b $3x - 2y = 4$ and $x + y = 8$

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c $5x - 7y = 3$ and $2x + 8y = 3$

d $-8x + 5y = 1$ and $3x + 18y + 7 = 0$

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