Bridging Unit: Polynomials and the binomial theorem

Topic B: Algebraic division



When simplifying fractions, divide the numerator and denominator by their highest common factor.

For example, $\frac{112}{140}$ can be simplified to its equivalent fraction $\frac{4}{5}$ by dividing numerator and denominator by their HCF: 28, which is the product of their common prime factors 7, 2 and 2.

Algebraic fractions can be simplified in the same way. You must first factorise the numerator and the denominator, then divide both the numerator and denominator by their highest common factor.

For example, $\frac{x(x+1)(x-2)}{x^2(x+1)} = \frac{x-2}{x}$ since the common factor of x(x+1) can be cancelled.

xample 1

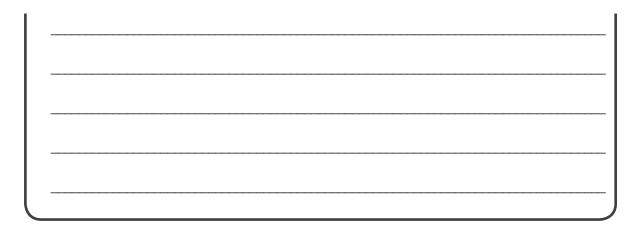
Simplify the fractions a $\frac{x^2 + 5x + 6}{2x^2 + 6x}$ b $\frac{2x^2 - 3x - 9}{4x^2 - 9}$	Factorise numerator and denominator.
a $x^2 + 5x + 6 = (x+2)(x+3)$ • $2x^2 + 6x = 2x(x+3)$ •	Cancel the common factor of $x + 3$ from numerator and denominator.
So $\frac{x^2 + 5x + 6}{2x^2 + 6x} = \frac{(x+2)(x+3)}{2x(x+3)} = \frac{x+2}{2x}$	Write $-3x$ as $-6x + 3x$ since $3 \times (-6) = -18$ and $2 \times (-9) = -18$
b $2x^2 - 3x - 9 = 2x^2 - 6x + 3x - 9$ = $2x(x-3) + 3(x-3)$ •	Factorise in pairs.
$= (2x+3)(x-3)$ $4x^2-9=(2x+3)(2x-3)$	Recognise the denominator as a difference of two squares.
So $\frac{2x^2 - 3x - 9}{4x^2 - 9} = \frac{(2x + 3)(x - 3)}{(2x + 3)(2x - 3)} = \frac{x - 3}{2x - 3}$	Cancel common factor 2x + 3 from

Simplify the fractions	$\mathbf{a} \; \frac{x(x+2)(x-1)}{x^2(x-1)}$	b $\frac{3x^2+2x-1}{x^2-1}$	Try It 1









If you can't simplify by cancelling common factors, you can use the method of **long division**. To use long division to divide the number 813 by 7

- See how many times 7 goes into 8, ignoring remainders: only once, so write 1 in the hundreds column.
- Then multiply 7 by this result and write it under 813: $7 \times 1 = 7$
- Subtract this number from the 8 above it to get a remainder of 1, and write that underneath again. Then copy down the 1 in the tens column.
- Repeat this process with the 11: again, 7 goes into 11 once, so write a 1, this time in the tens column.
- $7 \times 1 = 7$, so write a 7 underneath, and 11 7 = 4, so write a 4 underneath that. Then copy down the 3
- Finally, 7 goes into 43 six times, so write a 6 in the units column, and $7 \times 6 = 42$, so write 42 underneath.
- 43-42=1, so write 1 under that.
- To complete the solution, divide this 1 by the original divisor (7) to get your remainder: $813 \div 7 = 116 \frac{1}{7}$

You can use long division in the same way to divide polynomials. For example, to work out $(6x^2 + x - 12) \div (2x + 3)$

- First divide $6x^2$ by 2x to give 3x, and write this in the 'x-column'.
- Multiply 3x by 2x + 3 to get $6x^2 + 9x$ and write this underneath
- Subtract this from $6x^2 + x$ to get -8x and write that underneath again, then carry down the -12
- 2x goes into -8x negative four times, so write -4 in the next column. Then multiply this by 2x + 3 to get -8x - 12 and write that underneath.
- Subtracting these two rows gives 0, so the answer is 3x 4 exactly (if there had been a remainder, you would need to divide it by 2x + 3)

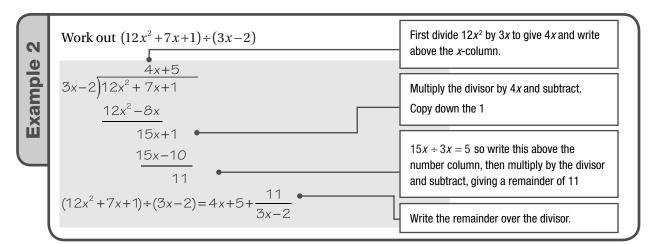
$$\begin{array}{r}
3x -4 \\
2x+3 \overline{\smash)6x^2 + x -12} \\
\underline{6x^2 +9x} \\
-8x -12 \\
\underline{-8x -12} \\
0
\end{array}$$

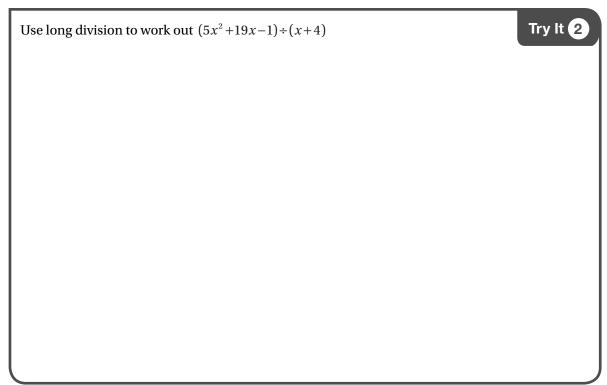




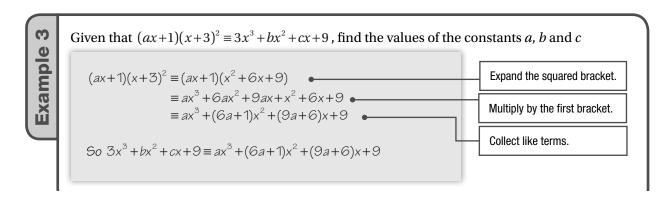
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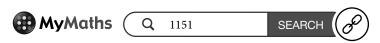


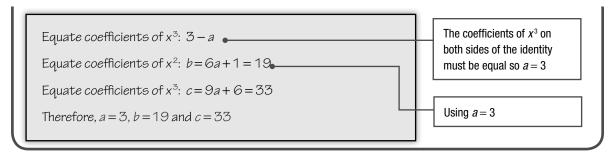


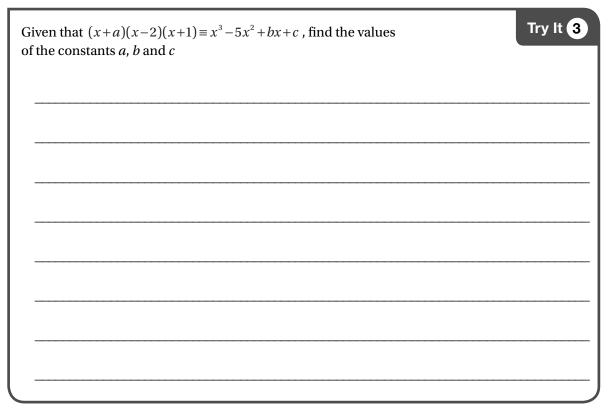


Another method of dividing polynomials involves **comparing the coefficients**. The **coefficient** is the number or constant that multiplies the variable. For example, for $2x^3 - 5x$, the coefficients of x^3 and x are 2 and -5. You can equate coefficients of the same variable from both sides of an identity.





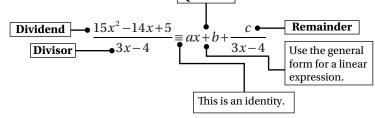




You can use the comparing coefficients technique and your knowledge of indices to divide algebraic expressions. As you saw in Example 2, dividing a quadratic polynomial by a linear polynomial gives a linear quotient and a constant remainder.

For example, for certain constants a, b and c To find the values of a, b and c you multiply both sides of the identity by 3x-4

$$15x^{2} - 14x + 5 \equiv (ax+b)(3x-4) + c$$
$$\equiv 3ax^{2} + (3b-4a)x - 4b + c$$



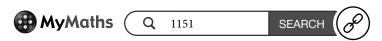
Comparing coefficients of x^2 gives $3a = 15 \implies a = 5$

Comparing coefficients of x gives $-14=3b-4a \implies 3b=4a-14=6 \implies b=2$

Comparing constant terms gives $5 = -4b + c \implies c = 5 + 4b = 13$

Therefore,
$$(15x^2 - 14x + 5) \div (3x - 4) = 5x + 2 + \frac{13}{3x - 4}$$

The same method can be used for higher-order polynomials, for example, dividing a cubic polynomial by a linear polynomial will give a quadratic quotient and a constant remainder.



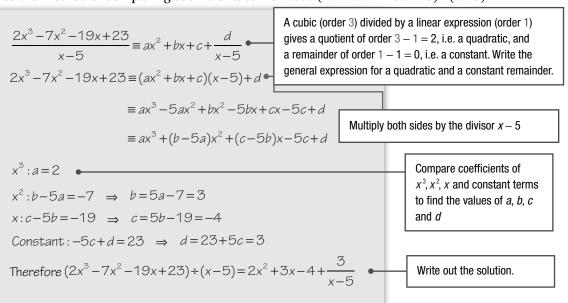
Dividing a polynomial of order m by a polynomial of order n where $m \ge n$ will give

- A quotient of order m-n
- A remainder of order at most n-1

For example, dividing two quadratics will give a constant quotient plus a linear remainder.

xample 4

Use the method of comparing coefficients to work out $(2x^3 - 7x^2 - 19x + 23) \div (x - 5)$



Use the method of comparing coefficients to work out $(6x^2 + 11x - 49) \div (3x - 5)$	Try It 4









Bridging Exercise Topic B

1 Simplify these fractions.

a
$$\frac{x(x-5)(x+2)}{x^3(x+2)}$$

b
$$\frac{(x+3)^2}{x(x+3)}$$

c
$$\frac{(x-4)}{2x(x-4)}$$

$$\mathbf{d} = \frac{x^2(x+5)}{x(x+5)^2}$$

2 Simplify these fractions by first factorising the numerator and the denominator.

a
$$\frac{x^2 - 2x - 8}{x^2 + 4x + 4}$$

 $\mathbf{b} \quad \frac{x^2 - 10x + 21}{x^2 - x - 6}$

 $\mathbf{c} \quad \frac{x^2 - 3x - 10}{x^2 - 10x + 25}$

d	$x^2 + 10x + 24$
u	2x + 8

$$e \frac{x^2+6x}{x^2-36}$$

$$\mathbf{f} = \frac{3x^2 + 6x}{x^2 - 5x - 14}$$

h	$x^2 - 64$	
"	$3x^{2}-24x$	

i
$$\frac{25-x^2}{45-4x-x^2}$$

:	$2x^2 - x - 28$
J	$2x^3 + 7x^2$

$$\mathbf{k} \quad \frac{15x^2 + 7x - 4}{10x^2 + 13x + 4}$$

$$I = \frac{x^3 - 100x}{6x^2 + 56x - 40}$$

m	$\frac{12x^3 + 36x}{2x^2 + 6}$	

n	$\frac{42x^2 - x - 1}{36x^2 - 12x + 1}$	

n	$9x^2 - 34x - 8$
þ	$2x^4 - 8x^3$

3 Use long division to work out these expressions.

a
$$(2x^2-9x-16)\div(x-6)$$

b	$(6x^2+3x+2)\div(x-1)$)

$(5x^2 +$	$41x+41) \div (x+7)$			
$(6x^2 +$	$-x-2)\div(2x-1)$			
15 r ² -	$+26x+5)\div(3x+4)$			
101	. 20% + 0 j + (0% + 4)			

$(8x^2+6x)$	-34)÷ $(4x+$	-9)			
$(3x^3+18)$	$3x^2 + 9x + 19$	$\div(x+5)$			



4 Find the values of the constants *a*, *b* and *c* in each of these identities.

a
$$(x+3)(x-4) \equiv ax^2 + bx + c$$

$$\mathbf{b} \quad (x-9)^2 \equiv ax^2 + bx + c$$

c
$$(x+4)(x+2)(x-2) \equiv x^3 + ax^2 + bx + c$$

$(x+5)^2(x+8) \equiv x^3 + ax^2 + bx + c$
$(ax+1)(x+3)(x-7) \equiv 5x^3 + bx^2 + cx - 21$
$(x+a)(x-1)(x+5) \equiv x^3 + bx^2 + cx + 10$

g $9x^3 + bx^2 + cx - 2 \equiv (ax+1)^2(x-2)$) where $a > 0$
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h
$$(x+a)(x+b)(x-4) \equiv x^3 - 8x^2 - 5x + c$$
 where $b > 0$

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5 Use the method of comparing coefficients to work out each of these divisions.

a
$$(3x^2-19x-18)\div(x-7)$$

b	$(4x^2+27x+20)\div(x+6)$
С	$(18x^2+3)\div(3x+1)$

d $(28x^2-120x-34)\div(2x-9)$	
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6 Use the method of comparing coefficients to simplify each of these fractions.

a
$$\frac{3x^3-2x^2-3x+6}{x-1}$$

x-1	

h	$14x^3 + 27x^2 + 7x - 4$
IJ	2 2 1 2

$$\mathbf{c} \quad \frac{3x^3 + 7x^2 - 11x + 4}{x^2 + 3x - 2}$$

$$\mathbf{d} = \frac{2x^3 - x - 1}{2x^2 - 4}$$

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